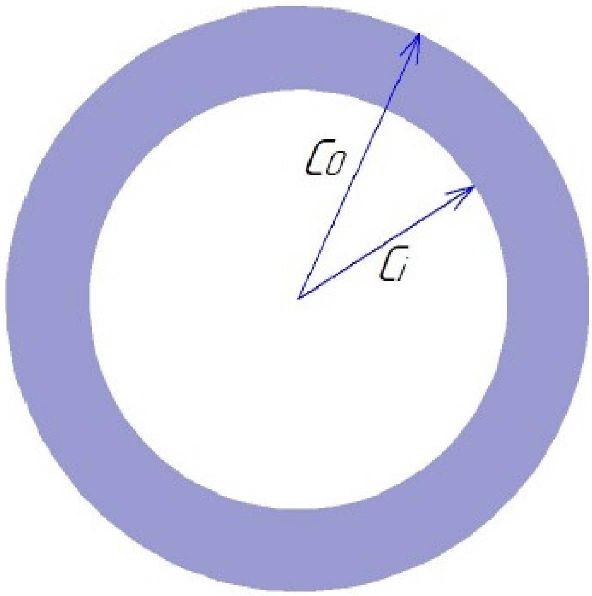
Engineering: Advanced Strength of Materials

Task #1:

Using an elastic-perfectly plastic material for the hollow shaft (see figure below), determine the minimum applied torque moment that causes yielding of the inner surface. Also, determine the angle of twist and the shear strain.



Solution:

When the yielding starts, the shear stress has a value of yielding stress, i.e.:

$$\tau = \tau_Y$$

Since the material is elastic-perfectly plastic, the yielding shear stress is constant:

$$\tau = \tau_Y = const$$

Hence, the torque moment can be written as:

$$T_P = 2\pi \cdot \tau_Y \int_{c_i}^{c_0} \rho^2 d\rho = 2\pi \cdot \tau_Y \left(\frac{\rho^3}{3} \right) \Bigg|_{c_i}^{c_0} = \frac{2}{3} \pi \cdot \tau_Y \left(c_0^3 - c_i^3 \right)$$

The angle of twist:

$$\gamma_Y = \frac{\tau_Y}{G}$$

$$\phi = \frac{\gamma_Y}{\rho_Y} L = \frac{\tau_Y / G}{c_i} L = \frac{\tau_Y L}{c_i G}$$

The shear strain:

$$\frac{\gamma_{\max}}{c_0} = \frac{\gamma_Y}{c_i}$$

$$\gamma_{\max} = \left(\frac{c_0}{c_i} \right) \tau_Y / G = \frac{c_0 \tau_Y}{c_i G}$$

Task #2:

Compare the value of maximum shear stress between a circular and rectangular shaft if they are made from the same material of 304 stainless steel, and they have the same area of 9 in^2. The torque moment is given to be 4000 lb^{x}in.

Solution:

For a circular shaft:

$$A = \pi c^2 = 9 \text{, then } c = \left(\frac{9}{\pi}\right)^{\frac{1}{2}} (in)$$

$$(\tau_c)_{\max} = \frac{T \cdot c}{J} = \frac{T \cdot c}{\frac{\pi}{2} \cdot c^4} = \frac{2T}{\pi \cdot c^3} = \frac{2 \times 4000}{\pi \left(\frac{9}{\pi}\right)^{\frac{3}{2}}} = 525(psi)$$

For a rectangular shaft:

$$A = a^2 = 9 \text{, then } a = (9)^{\frac{1}{2}} = 3(in)$$

$$(\tau_r)_{\max} = \frac{T \cdot a}{2J} = \frac{T \cdot a}{2\, a^4 / 12} = \frac{6T}{a^3} = \frac{6 \times 4000}{3^3} = 889(psi)$$

As can be seen, the rectangular cross section has a much larger (x 1.7) shear stress compared to a circular.